Probabilistic Sensitivity Analysis for Seismic Risk Based on Stochastic Sampling and Focusing on Stochastic Ground Motion Modeling

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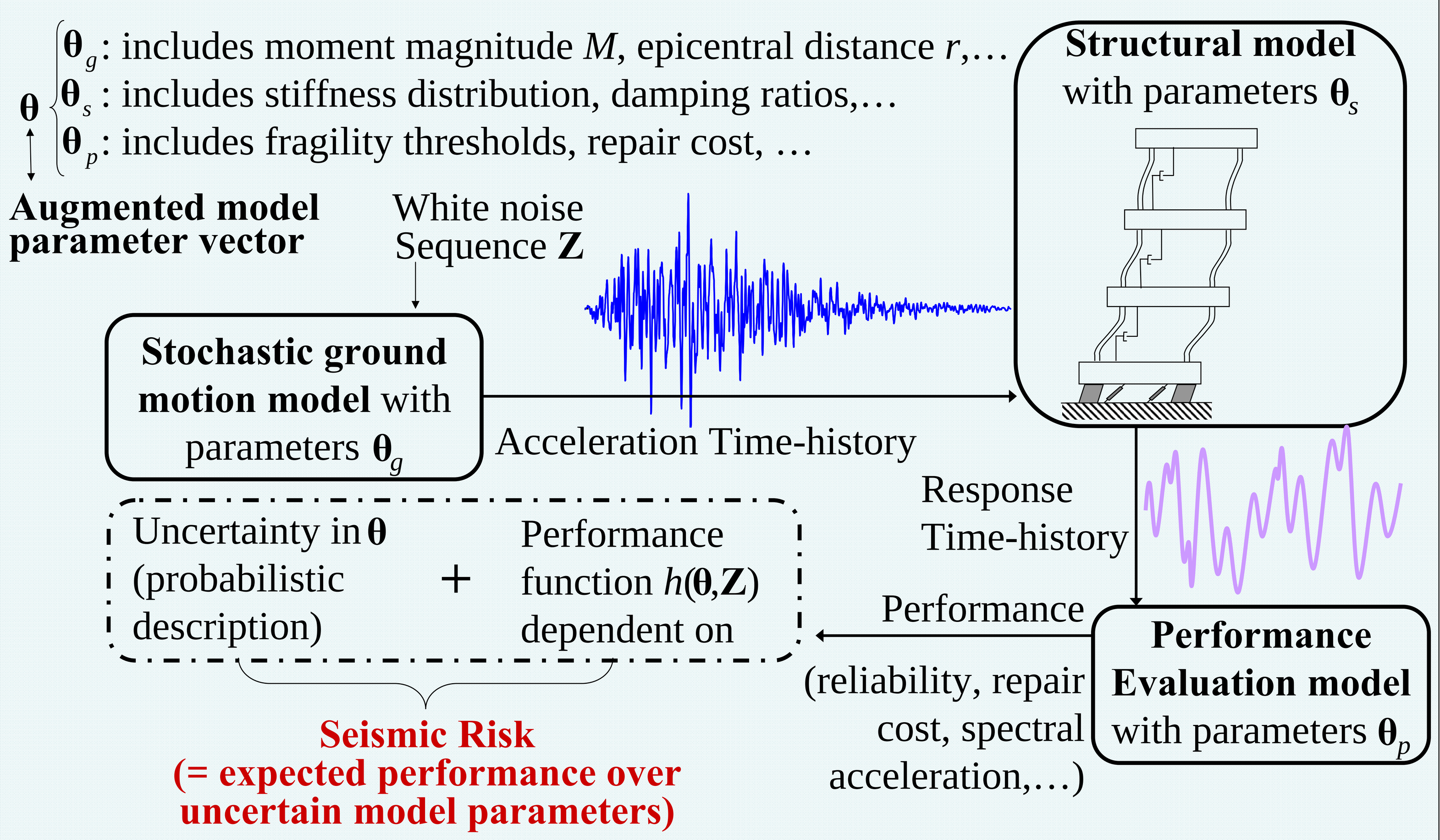
1. Motivation

Seismic risk assessment requires description of the time-history for future earthquakes. Stochastic ground motion models, either "source-based" or "record-based", are emerging as an attractive approach for such a task. Description of the uncertainty in the regional seismicity and in the predictive relationships connecting seismological characteristics to ground motion properties (such as strong motion duration or frequency content) facilitates then a complete probabilistic characterization for the seismic hazard and an augmented framework for seismic risk description



Probabilistic sensitivity analysis identifies which of the uncertain model parameters have higher contribution to the overall seismic risk

2. Objectives

Develop an efficient, simulation-based framework for evaluating seismic-risk sensitivity to different stochastic ground motion characterizations, to the probability models selected to describe the uncertainty related to this modeling process, and to the predictive relationships connecting them to the regional seismic hazard.

Fundamental questions to be answered are

- How can sensitivity be quantified based on information entropy principles and efficiently evaluated? How can the correlation between model parameters in affecting seismic risk be characterized? How can these tasks be efficiently performed for complex seismic hazard applications?
- What models characteristics are more important for affecting seismic risk?
- Which excitation properties impact this risk more?
- Are there fundamental differences between source-based and record-based stochastic ground motion models in describing the seismic hazard?
- How much do the answers in the previous questions change for different applications (different structural models) and different performance quantifications?
- What is the dependency on the specific probability models chosen to describe uncertainties?

The answers to these questions will lead to practical guidelines for ground motion modeling/selection and for probabilistic description of seismic hazard for a wide range of structural engineering applications

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3. Innovative Sensitivity Description

Motivated by definition of seismic-risk (i.e., stochastic integral)

$$H=\int h(\boldsymbol{\theta},\mathbf{Z})p(\boldsymbol{\theta})p(\mathbf{Z})d\boldsymbol{\theta}d\mathbf{Z}$$

Define auxiliary probability density function (proportional to integrand)

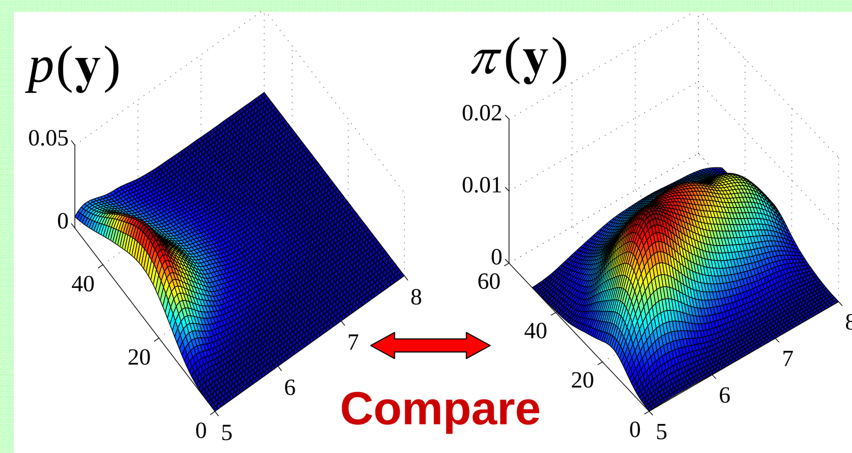
$$\pi(\boldsymbol{\theta},\mathbf{Z})\propto h(\boldsymbol{\theta},\mathbf{Z})p(\boldsymbol{\theta})p(\mathbf{Z})$$

For any group of model parameters y consider marginal distribution

$$\pi(\mathbf{y})\propto p(\mathbf{y})\int h(\boldsymbol{\theta},\mathbf{Z})p(\mathbf{x}\mid\mathbf{y})d\mathbf{x}$$

$$\mathbf{x}=\{\boldsymbol{\theta},\mathbf{Z}\}-\mathbf{y}$$

Comparison between $\pi(\mathbf{y})$ and $p(\mathbf{y})$ expresses sensitivity of seismic risk with respect to subset y. Bigger differences correspond to greater impact in the seismic risk



Quantify difference by relative information entropy

$$D(\pi(\mathbf{y})\,\|\,p(\mathbf{y}))=\int\pi(\mathbf{y})\log\left(\frac{\pi(\mathbf{y})}{p(\mathbf{y})}\right)d\mathbf{y}$$

Calculating and comparing $D(.)$ for different subsets reveals relative importance of each. The major challenge is the efficient calculation of marginal distribution $\pi(\mathbf{y})$ for the different subsets

4. Efficient Entropy Calculation

Case 1. Large dimensional white noise sequence: stochastic simulation

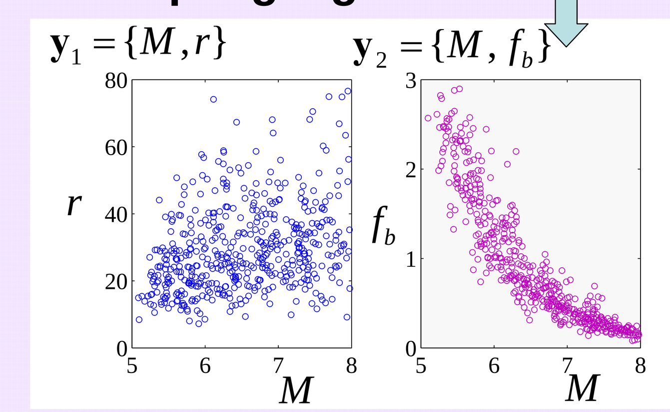
$$D(\pi(\mathbf{Z})\,\|\,p(\mathbf{Z}))\approx\frac{1}{n_{sz}}\sum_{j=1}^{n_{sz}}\frac{\hat{h}(\mathbf{Z}_j)}{\hat{H}}\log\left(\frac{\hat{h}(\mathbf{Z}_j)}{\hat{H}}\right);\ \mathbf{Z}_j\sim p(\mathbf{Z}),\boldsymbol{\theta}_i\sim p(\boldsymbol{\theta})$$

$$\hat{h}(\mathbf{Z}_j)=\frac{1}{n_{s\theta}}\sum_{i=1}^{n_{s\theta}}h(\boldsymbol{\theta}_i,\mathbf{Z}_j)\qquad\hat{H}=\frac{1}{n_{s\theta}n_{sz}}\sum_{i=1}^{n_{s\theta}}\sum_{j=1}^{n_{sz}}h(\boldsymbol{\theta}_i,\mathbf{Z}_j)$$

Case 2. Low dimensional group y: stochastic sampling

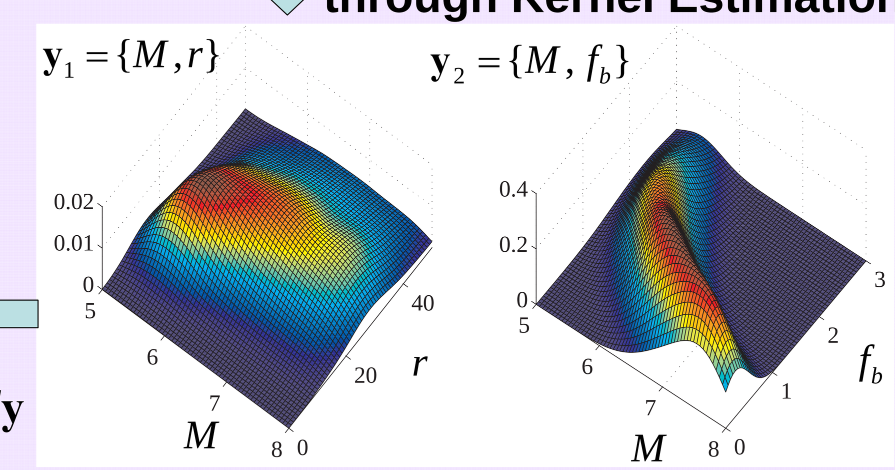
Step 1: Generate one large set of samples of $\{\boldsymbol{\theta},\mathbf{Z}\}$ from $\pi(\boldsymbol{\theta},\mathbf{Z})$ by some stochastic sampling algorithm

Step 2: Take projection to different subsets $\mathbf{y}_i$



[Computationally expensive step (step 1) is performed only once, and information for all interested subsets is obtained highly efficiently]

Step 3: Approximate $\pi(\mathbf{y})$ through Kernel Estimation

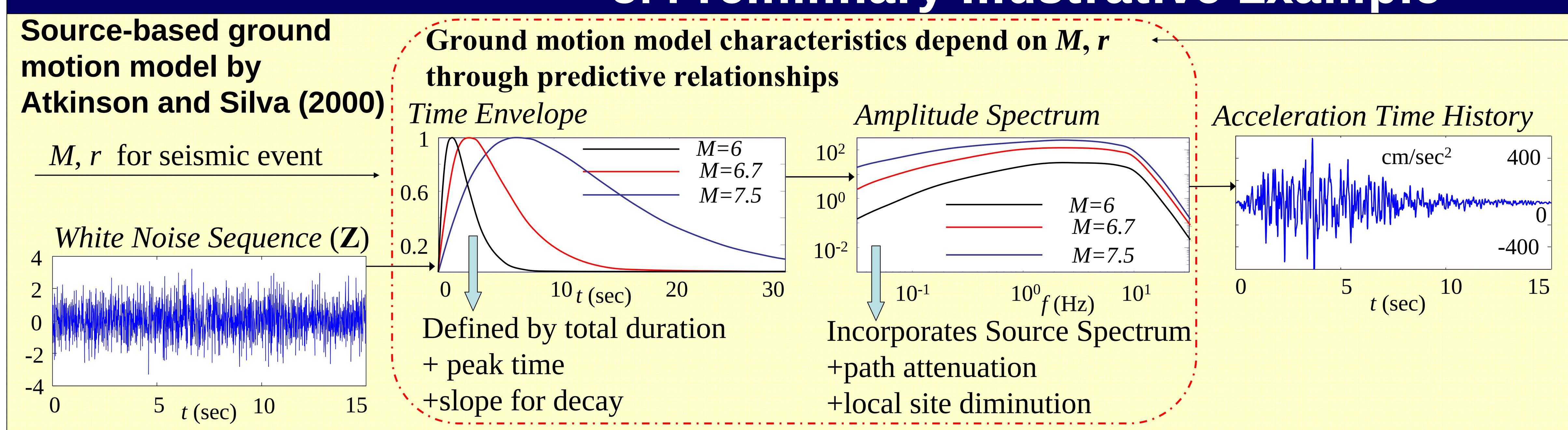


Step 4: Use approximation for $\pi(\mathbf{y})$ to calculate Entropy

$$D(\pi(\mathbf{y})\,\|\,p(\mathbf{y}))\approx\int\tilde{\pi}(\mathbf{y})\log\left(\frac{\tilde{\pi}(\mathbf{y})}{\tilde{p}(\mathbf{y})}\right)d\mathbf{y}$$

5. Preliminary Illustrative Example

Source-based ground motion model by Atkinson and Silva (2000)



Some Model Characteristics

$$\text{Source Spectrum}=\left[\frac{1-e}{1+(f/f_a)^2}+\frac{e}{1+(f/f_b)^2}\right]$$

$$\log_{10}f_a=2.181-0.496M\quad+e_a$$

$$\log_{10}f_b=2.41-0.408M\quad+e_b$$

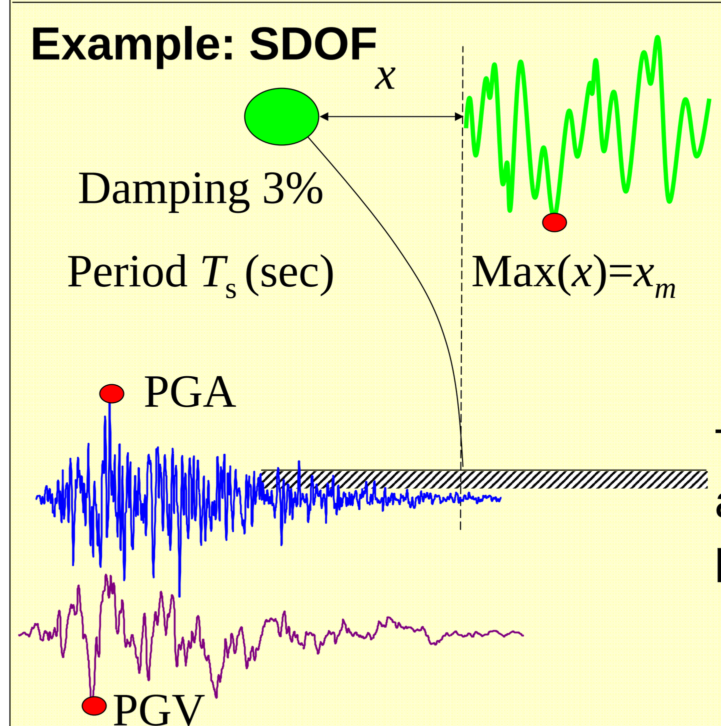
$$\log_{10}e=0.605-0.255M\quad+e_e$$

$$\text{Duration}=T_w$$

$$\log(T_w)=\log\left(\frac{1}{f_a}+\frac{\sqrt{r^2+10^{3-1M}}}{10}\right)+e_t$$

Uncertainty in predictive relationships (zero mean Gaussian error)

Example: SDOF

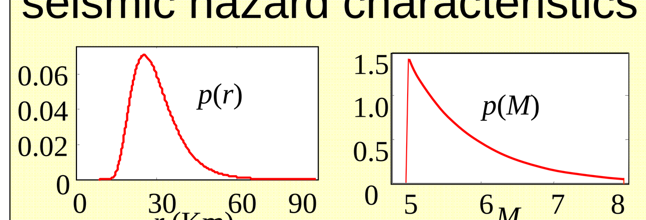


Risk quantifications

- Average PGA (--)
- Average PGV (⋯)
- Average x_m (—)
- Probability [PGA>b_a](--)
- Probability [x_m>b_x] (—)

Thresholds b_a and b_x adaptively selected so that probability is 1% (rare event)

Uncertainty considered for seismic hazard characteristics



and for all ground motion predictive relationships (coefficient of variation γ)

Preliminary result show (i) dominance of M in affecting seismic risk, (ii) strong dependency on exact risk quantification and on properties of SDOF, (iii) significant changes as level of uncertainty changes, and (iv) minor only importance of Z